9.2 ARITHMETIC SEQUENCES + SERIES

eg. 1, 3, 5, 7, 9, 11, 13, ...

+2 +2 +2 +2 +2 +2

$d = 2$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)(2)$

$a_n = 1 + 2(n-1)$

IF $a_3 = 11$ AND $a_8 = -4$, FIND a_{11} IF a IS AN ARITHMETIC SEQUENCE

$a_{11} = a_1 + (11-1)d = a_1 + 10d$

$11 = a_3 = a_1 + (3-1)d = a_1 + 2d \rightarrow a_1 + 2d = 11$ ①

$-4 = a_8 = a_1 + (8-1)d = a_1 + 7d \rightarrow a_1 + 7d = -4$

$-1 * ①$ $-a_1 - 2d = -11$

$5d = -15$

$d = -3$

$a_1 + 2(-3) = 11$

$a_1 - 6 = 11$

$a_1 = 17$

$a_{11} = a_1 + 10d$

$= 17 + 10(-3)$

$= 17 + -30$

$= -13$

$1, 3, 5, 7, 9, 11$ ← ARITHMETIC SEQUENCE

$1 + 3 + 5 + 7 + 9 + 11$ ← " SERIES

$$S_6 = 1 + 3 + 5 + 7 + 9 + 11$$
$$+ S_6 = (11 + 9 + 7 + 5 + 3 + 1)$$
$$2S_6 = 12 + 12 + 12 + 12 + 12 + 12 = 6 * 12$$

$$S_6 = \frac{6 * 12}{2} = 36$$

$1+11$ $3+9$

$= (1+2)+(11-2)$

$= 1+11$

$$S_n = a_1 + a_2 + a_3 + \cdots + a_{n-2} + a_{n-1} + a_n$$
$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-3)d) + (a_1 + (n-2)d) + (a_1 + (n-1)d)$$
$$S_n = (a_1 + (n-1)d) + (a_1 + (n-2)d) + (a_1 + (n-3)d) + \cdots + (a_1 + 2d) + (a_1 + d) + a_1$$

$$2S_n = (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + \cdots + (2a_1 + (n-1)d)$$

$$2S_n = n(2a_1 + (n-1)d)$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

OR $S_n = \frac{n}{2}(a_1 + a_n)$

OR $S_n = \frac{a_1 + a_n}{2} n$

SANITY CHECK:

$S_6 = \frac{6}{2}(2 \cdot 1 + 5 \cdot 2) = 3(2 + 10)$

$= 36$ ✓

$a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}(2a_1 + (n-1)d) = \frac{n}{2}(a_1 + a_n)$

FIND THE SUM OF THE FIRST 100 TERMS OF THE ARITHMETIC SEQUENCE WITH FIRST TERM -15 AND 8^{TH} TERM 6.

$$S_{100} = \frac{100}{2}(2(-15) + (100-1)d)$$
$$= 50(-30 + 99 \cdot 3)$$
$$= 50(-30 + 297)$$
$$= 50(267)$$
$$= 13350$$

$a_8 = a_1 + (8-1)d$

$6 = -15 + 7d$

$21 = 7d$

$d = 3$

JOE STARTED A NEW JOB ON JAN 1, 2007 AT A STARTING SALARY OF $\$57,000$

HE RECEIVED EQUAL SIZED RAISES EACH JAN 1ST

SALARIES FORM ARITHMETIC SEQUENCE

HE LEFT ON DEC 31, 2015

IF HIS SALARY ON JAN 1, 2010 WAS $\$71,000$,

(1) WHAT WAS HIS SALARY WHEN HE LEFT?

(3) HOW MUCH DID HE EARN ALTOGETHER AT THAT JOB?

LET a_n = JOE'S SALARY ON JAN 1 OF HIS n-TH YEAR

a_1 = " " " " " 1ST YEAR = 57,000 (2007)

$a_4 = 71{,}000$ $\quad$ 2010 − 2007 = 3

2010 − 2007 + 1 = 4

$a_n = a_1 + (n-1)d$

(1) WANT a_9 = SALARY ON DEC 31, 2015

= SALARY ON JAN 1, 2015

2015 − 2007 + 1 = 9

$a_9 = a_1 + 8d$

$a_9 = 57{,}000 + 8\left(\frac{14000}{3}\right)$

$= 57{,}000 + \frac{8 \cdot 14{,}000}{3}$

$a_4 = a_1 + 3d$

$71{,}000 = 57{,}000 + 3d$

$14{,}000 = 3d$

$d = \frac{14{,}000}{3}$

(2) WANT $S_9 = \frac{9}{2}\left(2(57{,}000) + 8\left(\frac{14{,}000}{3}\right)\right)$